

Possible triplet superconductivity in MOSFETs

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A theory that predicts a spin-triplet, even-parity superconducting ground state in two-dimensional electron systems is re-analyzed in the light of recent experiments showing a possible insulator-to-conductor transition in such systems. It is shown that the observations are consistent with such an exotic superconductivity mechanism, and predictions are made for experiments that would further corroborate or refute this proposal.

Since 1979 it had been generally believed that two-dimensional (2- d) electron systems cannot undergo a true metal-insulator transition (MIT), but rather are always insulating at zero temperature ($T = 0$), even for arbitrarily weak disorder. Initially, this was the conclusion only for models of noninteracting electrons,¹ but later developments²⁻⁴ strongly suggested that it remains valid in the presence of electron-electron interactions. Recent experiments on Si MOSFETs have challenged this conventional wisdom.⁵⁻⁸ The samples used in these experiments are in an unprecedented parameter regime, as they achieve high electron mobilities at low electron densities. The apparent quantum phase transition from an insulating phase to a conducting one that is observed in these samples occurs at very low densities, which leads to a very strong effective electron-electron interaction. The disorder is also very strong, as is indicated by a ‘separatrix’ value of the sheet resistance that is about three times the Mott number. A MIT has also been observed in p-type SiGe quantum wells with parameter values that are very close to those in the MOSFETs.⁹

These observations can in principle be theoretically explained in a number of ways. For example, it is known that there is a MIT in 2- d systems with spin-orbit scattering and either no interaction, or a short-ranged interaction between the electrons.⁴ While this universality class has been invoked in the context of the experiments in question,¹⁰ it is unclear why it should apply to the systems under consideration, where the electrons interact by means of a strong, long-ranged Coulomb interaction. Other possible theoretical explanations include superconductor-to-insulator transitions (SITs), which are well known to occur in $d = 2$.⁴ Indeed, it has been noted in Refs. 6 and 7 that the observed transition has many features in common with SITs. It has also been pointed out in Refs. 11 and 12 that the conducting phase is likely to be a superconductor, or a perfect conductor, respectively.

Some time ago the present authors have discussed a mechanism that can lead to spin-triplet, even-parity superconductivity in disordered, interacting electron

systems.¹³ The predicted effect is strongest in $d = 2$, and MOSFETs were explicitly mentioned in Ref. 13 as potential realizations of this phenomenon. This motivates us to analyze this mechanism semi-quantitatively for parameter values that are appropriate for the new MOSFET samples. As we will show below, the observations are compatible with an explanation in terms of superconducting fluctuations that precede the predicted exotic superconducting state, and we are also able to make predictions for observables other than the resistivity.

The physical mechanism that underlies the exotic superconductivity discussed in Ref. 13 is based on the slow decay of charge and spin polarization clouds that is caused by long-time tail effects in disordered systems. The details have been explained in Ref. 13, and need not be repeated here. We just recall that there are two physically distinct mechanisms, based on charge and spin fluctuations, respectively. Characteristic features of the ensuing even-parity, spin-triplet superconductivity are, (1) a very low T_c for parameter values that are characteristic of conventional 2- d electron systems, (2) a very rapid variation of T_c as a function of parameter values such as the bare interaction amplitudes and the disorder, (3) a gapless tunneling density of states, and (4) a logarithmic dependence of the specific heat coefficient on the temperature. All of these features were derived and discussed in Ref. 13. In addition, one finds (5) an ordinary Meissner effect, like in a gapless BCS superconductor, as we will now proceed to show. To this end, we calculate the transverse current-current susceptibility, using the field-theoretic formulation of Ref. 13. A standard calculation at the same level as our previous saddle-point theory for the order parameter yields a transverse current susceptibility

$$\chi_T(\mathbf{k} \rightarrow 0) = \frac{-2}{dm^2} \frac{1}{V} \sum_{\mathbf{p}} \mathbf{p}^2 T \sum_n [\mathcal{G}_n(\mathbf{p}) \mathcal{G}_n(\mathbf{p}) - \mathcal{F}_n(\mathbf{p}) \mathcal{F}_{-n}(\mathbf{p})] \quad (1a)$$

Here \mathcal{G} and \mathcal{F} are the normal and anomalous Green functions, respectively,

$$\mathcal{G}_n(\mathbf{p}) = \frac{i\omega_n + \xi_{\mathbf{p}}}{\omega_n^2 + \xi_{\mathbf{p}}^2 - \Delta_n \Delta_{-n}} ,$$

$$\mathcal{F}_n(\mathbf{p}) = \frac{\Delta_n}{\omega_n^2 + \xi_{\mathbf{p}}^2 - \Delta_n \Delta_{-n}} , \quad (1b)$$

with ω_n a fermionic Matsubara frequency, $\xi_{\mathbf{p}} = \mathbf{p}^2/2m - \epsilon_F$ with ϵ_F the Fermi energy, and Δ_n the frequency dependent gap function. The gap function on the imaginary axis is purely real, $\Delta_n = -\Delta_{-n} = \Delta(i\omega_n) \equiv \delta(\omega_n)$, with $\delta(\omega)$ a real function.¹⁴ At zero temperature, Eq. (1a) can then be expressed as a frequency integral,

$$\chi_T(\mathbf{k} \rightarrow 0) = \frac{n}{m} \left[1 - \int_0^\infty d\omega \frac{\delta^2(\omega)}{(\omega^2 + \delta^2(\omega))^{3/2}} \right] , \quad (1c)$$

with n the electron number density. We see that the correction to the normal-metal result, n/m , is negative definite.¹⁵ Consequently, we have $\chi_T(\mathbf{k} \rightarrow 0) < n/m$, which guarantees ideal diamagnetism.

Si MOSFETs were discussed in Ref. 13 as possible realizations of the proposed mechanism, but for typical parameter values at best ultralow values for T_c , in the μK region, were to be expected. For the samples investigated in Refs. 5–8, the situation is very different. Let us estimate the mean-field T_c for these samples, assuming the charge fluctuation mechanism described in Ref. 13 (we will come back to the spin fluctuation mechanism later). The mean-field value of T_c is given by

$$T_c = T_F \frac{(\bar{\kappa}/k_F)^2}{\hat{R}_\square} \exp\left(\frac{-4\pi^2/\ln 2}{\hat{R}_\square}\right) . \quad (2)$$

Here $\bar{\kappa} = \kappa(1 + F_0^s)$, with κ the Thomas-Fermi screening wavenumber, and F_0^s a Landau parameter. T_F is the Fermi temperature, and \hat{R}_\square is the resistance per square measured in units of $\hbar/e^2 = 4108\Omega$. With an electron density $n = 10^{11} \text{ cm}^{-2}$ and a valley degeneracy $g_V = 2$, we have a Fermi wavenumber $k_F = 5.6 \times 10^5 \text{ cm}^{-1}$. With an effective mass $m^* = 0.2 m_e$, this corresponds to a Fermi temperature $T_F = 6.9 \text{ K}$. Assuming a Landau parameter $F_0^s = -0.9$, the screening wavenumber is $\bar{\kappa} = 1.5 \times 10^7 \text{ cm}^{-1}$. At a sheet resistance equal to $R = 0.8 \times 2\pi\hbar/e^2$, the T_c formula, Eq. (2), yields $T_c \approx 1.7 \times 10^{-3} T_F \approx 10 \text{ mK}$.

In the experimental range of Refs. 5–8, one therefore does not expect to see true superconductivity resulting from the charge fluctuation mechanism. If the observed transport anomalies are to be explained in terms of it, then they must be caused by superconducting fluctuations. Let us therefore estimate the width of the temperature region within which one expects substantial fluctuations. To this end we recall that the kernel in the gap or T_c equation in Ref. 13 takes the form of a function F , whose argument depends on T and \hat{R}_\square . For asymptotically small arguments, F approaches a logarithm, and we have

$$F(\hat{R}_\square(k_F/\bar{\kappa})^2 T/T_F) \approx \ln(\hat{R}_\square(k_F/\bar{\kappa})^2 T/T_F) \equiv f(T). \quad (3)$$

If we consider T the flow parameter in a RG description of the system, then \hat{R}_\square becomes T -dependent, and for fixed bare parameters F can be considered a function f of T , as is indicated in Eq. (3). If we replace T by T_c , and use the above parameter values, then we obtain $f(T_c) \approx -10$. As a criterion for noticeable fluctuations, we require the temperature to be lower than some T^* defined by $f(T^*) = -5$. As the temperature increases from T_c , the scale dependent quantity \hat{R}_\square increases, but in $d = 2$ it does so only logarithmically.⁴ The most important T -dependence is therefore the explicit one in Eq. (3). Keeping only this leading T -dependence, the estimate becomes structurally the same as in BCS theory. Using the same parameters as for our estimate of T_c above, we find

$$T^*/T_c \approx e^5 \approx 10^2 . \quad (4)$$

We also have obtained an estimate of the same order by using the RG flow equation for the disorder parameter,⁴ which determines the T -dependence of the resistivity.

With the above values for T_c and T^* , in a 2- d system with parameter values appropriate for the new MOSFETs, Eq. (4) predicts that observable superconducting fluctuations are to be expected up to temperatures of about 1K, and the fluctuations should be substantial at the lowest temperatures reached in the experiments. This could well explain the observations. We stress that these considerations amount at best to an order-of-magnitude estimate, while reliable calculations of the superconducting T_c are notoriously difficult for any mechanism. Nevertheless, the results show that our mechanism is a viable candidate for explaining the observations. It is also encouraging that our T_c is an extremely rapidly varying function of the parameter values. This may explain why similar effects were not observed in other 2- d systems.

A magnetic field weakens our superconductivity mechanism, since it quenches the strong renormalization of interaction parameters that leads to the pairing. In Ref. 13 we estimated the upper critical field to be $H_{c2} \approx (\hbar c/e) k_F^2 T_c/T_F$. At the fluctuation temperature $T^* > T_c$, one would then expect a field of strength $H^* = (\hbar c/e) k_F^2 T^*/T_F$ to have a qualitative effect on the transport properties. At $T^* = 1 \text{ K}$, this yields $H^* = 9 \text{ kOe}$. This is in very good agreement with the observed suppression of the conducting phase by a magnetic field.¹⁶

While the above estimates show that our proposed mechanism is definitely compatible with the existing experiments, other observations are needed to convincingly corroborate or refute our proposal. Of particular interest are the properties of the superconducting phase, if any, that we predict to exist at sufficiently low temperatures. Apart from the prediction of a Meissner effect, we have calculated three observables, viz. the gap function and

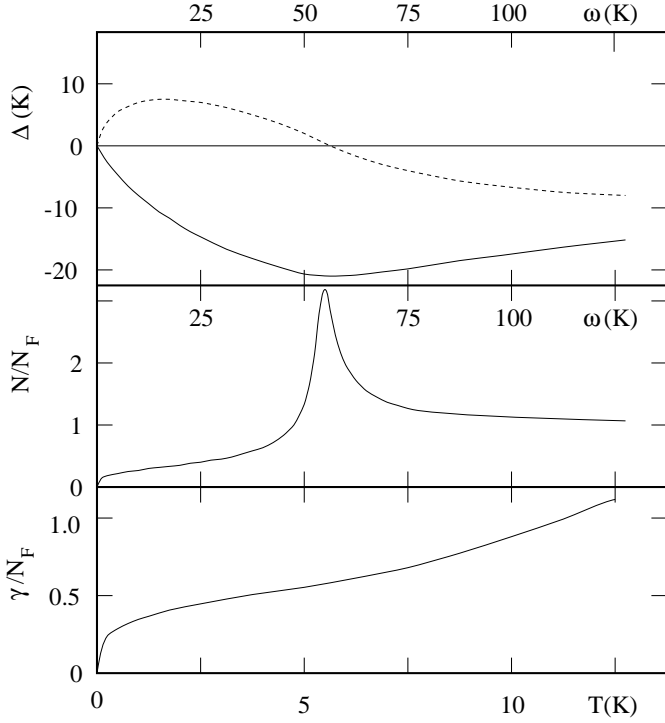


FIG. 1. Real part (solid line) and imaginary part (dashed line) of the gap function Δ at $T = 0$ as a function of the frequency ω , the tunneling density of states N at $T = 0$ as a function of the frequency, and the specific heat coefficient γ as a function of the temperature T as predicted by the theory for the parameter values discussed in the text. N and γ are normalized by the free electron density of states per spin, N_F , and Δ , ω , and T are measured in Kelvin.

the tunneling density of states, both at $T = 0$ as a function of the frequency, and the specific heat coefficient, $\gamma = C_V/T$, as a function of T , in the superconducting phase. Such calculations for any purely electronic superconductivity mechanism pose an intrinsic problem, namely that the feedback of the superconducting state on the coupling mechanism must be taken into account.^{17,13} This problem has so far not been solved. In the case of the present mechanism it is likely to just lead to a smaller effective coupling constant $\lambda = \hat{R}_\square/\pi^2$, i.e. a smaller \hat{R}_\square , then the one that enters the T_c equation, except for a qualitative effect in the asymptotically low frequency regime.¹³ For our present illustrational purposes we choose an effective coupling constant $\lambda = 0.4/\pi$, which is a factor of 4 smaller than what corresponds to the \hat{R}_\square used for the T_c estimate above. The results are shown in Fig. 1. Notice that the purely electronic pairing mechanism leads to a pronounced pseudo-gap structure on a frequency scale that is, for our value of the coupling constant, some 5,000 times larger than T_c . The T_c scale is reflected in the region of validity for the asymptotic low-frequency and low-temperature behavior.¹³ It turns out that the overall shapes of the curves do not

strongly depend on the effective λ chosen, only the energy scales do. The two relevant energy or frequency scales are¹³ $\Omega_1^* = (2 \times 10^{-2}/\lambda)(\hbar/k_F)^2 T_F$, which sets the overall frequency scale, and $\Omega_2^* = \Omega_1^* \exp(-1/\sqrt{\lambda})$, which separates the low and high-frequency regimes ($\Omega_{1,2}^*$ are the frequency scales $\omega_{1,2}^*$ of Ref. 13 multiplied by $\pi/100$). In conventional, phonon-induced, superconductivity the role of Ω_1^* is played by the Debye frequency, and that of Ω_2^* by the value of the gap function at zero frequency. For the parameter values chosen, the numerical values of the two frequency scales are $\Omega_1^* = 778$ K, and $\Omega_2^* = 47$ K.

Figure 1 shows our qualitative predictions for the superconducting state. For experiments *above* T_c , the most promising quantity to measure would probably be the tunneling density of states, which should reveal the development of the characteristic structure that is shown in Fig. 1. Although the full features would be present only in the superconducting state, one would expect some structure in the DOS in the region where the transport properties are strongly anomalous. This structure should become stronger with decreasing temperature, but not develop into a real gap, in contrast to a BCS superconductor, but rather into the pseudogap shown in Fig. 1.

We now briefly discuss the spin fluctuation mechanism, which in principle is capable of producing much higher values of T_c than the charge fluctuation mechanism. However, for technical reasons¹³ it is harder to make semi-quantitative predictions in this case. The T_c -formula derived in Ref. 13 is

$$T_c = T_0 \exp \left[\frac{-\pi^2}{\hat{R}_\square \gamma_t^0} \left(1 - \frac{1}{32} \hat{R}_\square \gamma_t^0 \right) \right], \quad (5)$$

Here γ_t^0 is a bare (i.e., high-temperature) spin-triplet interaction constant, and T_0 is a microscopic temperature scale that is on the order of the Fermi temperature. If we assume $\gamma_t^0 = 1$, which corresponds to a high-temperature spin susceptibility that is twice that of free fermions with mass m^* , and the disorder as above, then we obtain a value for T_c that is a substantial fraction of T_F . This is certainly an overestimate, for reasons discussed in Ref. 13. However, the point is that the spin fluctuation mechanism allows for much higher T_c -values than the charge fluctuation mechanism, provided that there are strong spin fluctuations in the system, i.e., a large magnetic susceptibility in the normal metal. If the spin susceptibility of the 2- d electrons in MOSFETs could be measured, then it would be very interesting to look for a correlation between an enhanced spin susceptibility and the occurrence of the metallic phase. The qualitative features of the other observables discussed above are virtually unchanged for the spin fluctuation mechanism. Explicit calculations¹³ of the gap function, the density of states, and the specific heat, yield results very similar to those shown in Fig. 1, but with the characteristic features occurring on a smaller frequency or temperature scale.

We conclude by means of two additional remarks. First, there has been a recent report about a similar

conductor-to-insulator transition in Si/SiGe heterostructures with a much smaller value for the disorder than in the MOSFETs discussed above.¹⁹ However, the interpretation of these data has been called into question,²⁰ and more experimental work seems to be needed in order to ascertain if the phenomenon indeed persists to such low disorder. It is worthwhile to point out that long-time tails exist even in clean Fermi liquids, and a mechanism for triplet superconductivity similar to the one discussed above is conceivable. The effect would be weaker, though, and it would probably not result in a superconducting ground state even in $d = 2$.

Second, there are other possible interpretations of the experiments, which are not related to superconductivity. For instance, it was pointed out in Ref. 21 that the RG flow equations for disordered interacting electrons in $d = 2$ contain a nonperturbative critical fixed point that corresponds to a MIT. Since the fixed point is separated from the weak disorder regime by a region of runaway flow, its physical significance was considered doubtful. However, Castellani et al.²² have recently speculated that this fixed point might describe a MIT in $d = 2$. In this context we point out that one way for the fixed point to attain a physical meaning would be to make physical sense out of the runaway flow, e.g. in terms of a non-Fermi liquid state at small disorder.²³ Indeed, in $d = 3$, these runaway flows have been related to the ferromagnetic state.²⁴ The authors of Ref. 22 have also argued that even in the absence of a true MIT, the known flow equations^{3,4} for disordered interacting electrons in $d = 2$ allow for metallic behavior over a sufficiently large temperature region to explain the experimental observations. This would require, however, a large spin susceptibility in the metallic phase, large enough to trigger superconductivity via the spin-triplet mechanism according to our estimates. Thus a combination of some of these suggestions is conceivable: The pre-asymptotic analysis of Ref. 22 could explain the existing experimental data, but at lower temperatures a transition to the superconducting state discussed in this paper could occur. In either case, experiments looking for an enhancement of the magnetic susceptibility would be very interesting.

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